PROPOSED TITLE OF THE RESEARCH

ON GROWTH PROPERTIES AND FIXED POINT THEORY OF ANALYTIC FUNCTIONS

By

Sumanta Ghosh

Registration No:-VB-1022 of 2017-18

Date of Registration :- 29.07.2017

Under the supervision of

Dr. Dibyendu Banerjee

Department of Mathematics

Siksha-Bhavana

Visva-Bharati

Synopsis

Keywords: Analytic functions, functions of class II, Fix point, Exact order, Factor order, Growth.

Let$f(z)$ be an entire function. Set $f_1(z) = f(z)$ and inductively for positive integer $n \geq 1$, $f_{n+1}(z) = f(f_n(z))$. The solution of the equation $f_n(z) = z$ are called fix-point of $f(z)$ of order $n(\geq 1)$. If α is a fix point of $f(z)$ of order $n(> 1)$, but no lower order , then α is called a fix point of exact order n. Regarding the existence of fix points Rosenbloom [6]proved that$f(z)$ always has fixed point of exact order 1 or 2. After the work of Rosenbloom , Baker [1] made further investigation and proved that if $f(z)$ is a transcendental entire function, then $f(z)$ possesses infinity of many fix points of exact order n, except at most one value of n.For a polynomial function, Baker[1] proved that if $f(z)$ has at least one fix point of exact order n, for every positive integer n, with at most one exception.Bhattacharyya [3] extended the above theorem of Baker on transcendental entire function to the function $f(z)$of class II.Bhattacharyya [3] considering only such normalised function, proved that $f(z)$ has an infinity fix points of exact order n, for every positive integer n.In 1960 ,I.N.Baker states that if $f(z)$ is an entire function other than a linear polynomial then there are fix points of exact order n of $f(z)$ except for at most one value of n. In 1997, Lahiri and Banerjee define a special type of iteration, called relative iteration.In 2010 Banerjee and Jana proved that if $f(z)$ and $\phi(z)$ are transcendental entire function, then there are relative fix points of order n of $f(z)$, except for at most one value of n.In 2010, Banerjee and Jana define another type of function called generalised composite entire function. Lahiri and Banerjee [5] proved that if $f(z)$ and $\phi(z)$ belong to class II,then $f(z)$ has an infinity of relative fix points of exact order n for every positive integer n, provided $\frac{T(r,\phi_n)}{T(r,f_n)}$ is bounded.In 2015,Banerjee and Mandal define relative iterations for three functions and extend the result of Banerjee and Mandal [2] in that direction.They introduced that a point α is said to be a fix point of $f(z)$ of order n with respect to $g(z)$ and $h(z)$, if $f_n(\alpha) = \alpha$ and a fix point of $f(z)$ of exact order n if $f_n(\alpha) = \alpha$ but $f_k(\alpha) \neq \alpha$ and $g_k(\alpha) \neq \alpha$ for all divisors $k(< n)$ of n. they proved that if $f(z), g(z)$, and $h(z)$ belomg to class II, then $f(z)$ has infinitely many fix points of

exact factor order n for every integer $n(\geq 3)$ provided $\frac{T(r,g_{n)}}{T(r,f_{n)}}$ and $\frac{T(r,h_n)}{T(r,f_n)}$ are bounded.Then Banerjee and Mandal proved that if $f(z)$ and $\phi(z)$ belong to class II, then $f(z)$ has an infinity of generalised fix points of exact factor order n, provided $\frac{T(r,\phi_n)}{T(r,f_n)}$ is bounded. We now introduced that a point α is called a fix point of $f(z)$ of order n with respect to $g(z), h(z)$ and $w(z)$,if $f_n(\alpha) = \alpha$ but $f_k(\alpha) \neq \alpha, g_k(\alpha) \neq \alpha, h_k(\alpha) \neq \alpha$ and $w_k(\alpha) \neq \alpha$, for all divisors $k(< n)$ of n. Our main theorem is that if $f(z), g(z), h(z)$ and $w(z)$ belong to class II,then $f(z)$ has infinitely many fix pointsof exact order n for every positive integer $n(\geq 4)$ provided $\frac{T(r,g_n)}{T(r,f_n)}, \frac{T(r,h_n)}{T(r,f_n)}$ and $\frac{T(r,w_n)}{T(r,f_n)}$ are bounded.

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